TRINITY COLLEGE

IN NOMINE DOMINI

Year 11 Mathematics Specialist
Test 2 2022

Section 1 Calculator Free
Vectors

STUDENT'S NAME MARKING KEY

DATE: Friday 1st April **TIME:** 30 minutes **MARKS**: 30

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Given $\underset{\sim}{a}=\begin{pmatrix}-2\\6\end{pmatrix}$ and $\underset{\sim}{b}=\begin{pmatrix}12\\x\end{pmatrix}$. Determine the value of x if $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular vectors.

$\underset{\sim}{a}\cdot\underset{\sim}{b}=0$

$\begin{pmatrix}-2\\6\end{pmatrix}\cdot\begin{pmatrix}12\\x\end{pmatrix}=0$ ✓

$-24+6x=0$ ✓

$x=4$ ✓

2. (9 marks)

Given the vectors $\underset{\sim}{a} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$, $\underset{\sim}{b} = -3\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{c} = x\underset{\sim}{i} - 5\underset{\sim}{j}$, determine the following:

(a) The exact magnitude of $\underset{\sim}{a}$ in simplest surd form. [2]

$|\underset{\sim}{a}| = \sqrt{16+4}$ ✓ $= 2\sqrt{5}$ ✓

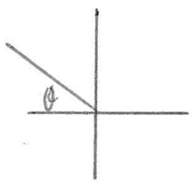
$= \sqrt{20}$

(b) Vector $\underset{\sim}{d}$, which is a vector twice as long as $\underset{\sim}{c}$, but in the opposite direction. [2]

$\underset{\sim}{d} = -2(x\underset{\sim}{i} - 5\underset{\sim}{j})$ $2\begin{pmatrix} -x \\ 5 \end{pmatrix}$

$= -2x\underset{\sim}{i} + 10\underset{\sim}{j}$ ✓✓

(c) The angle that $\underset{\sim}{b}$ makes with the positive x axis. [2]



$\tan\theta = \frac{3}{3}$ $\therefore$ angle is $135°$ ✓

$\theta = 45°$ ✓

(d) $\underset{\sim}{e}$, given that $\underset{\sim}{e} = 4\underset{\sim}{a} - \underset{\sim}{b}$ [2]

$\underset{\sim}{e} = 4\begin{pmatrix} 4 \\ -2 \end{pmatrix} - \begin{pmatrix} -3 \\ 3 \end{pmatrix}$ ✓

$\underset{\sim}{e} = \begin{pmatrix} 19 \\ -11 \end{pmatrix}$ ✓

(e) $\hat{\underset{\sim}{a}}$, a unit vector in the same direction as $\underset{\sim}{a}$. [1]

$\hat{\underset{\sim}{a}} = \frac{1}{2\sqrt{5}}\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ ✓

3. (10 marks)

(a) Given that $|\underset{\sim}{a}| = 4$, $|\underset{\sim}{b}| = 3$ and $\underset{\sim}{a} \cdot \underset{\sim}{b} = -6$

(i) Determine the size of the angle between vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$. [2]

$$\cos\theta = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} \qquad \theta = 120^\circ$$

$$= \frac{-6}{12}$$

(ii) Determine the exact value of $|\underset{\sim}{a} - \underset{\sim}{b}|$. [3]

$$\text{if } \underset{\sim}{a} \cdot \underset{\sim}{a} = |\underset{\sim}{a}|^2$$

$$(\underset{\sim}{a} - \underset{\sim}{b}) \cdot (\underset{\sim}{a} - \underset{\sim}{b}) = |\underset{\sim}{a} - \underset{\sim}{b}|^2 \checkmark$$

$$\underset{\sim}{a} \cdot \underset{\sim}{a} - 2\underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{b}$$

$$4^2 - 2(-6) + 3^2 \quad \checkmark \qquad = \sqrt{37} \checkmark$$

(b) Given $\underset{\sim}{c} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ and $\underset{\sim}{d} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$. Determine the vector projection of $\underset{\sim}{d}$ onto $\underset{\sim}{c}$. [4]

$$\text{proj}_{\underset{\sim}{c}} \underset{\sim}{d} = (\underset{\sim}{d} \cdot \hat{\underset{\sim}{c}}) \times \hat{\underset{\sim}{c}}$$

$$|\underset{\sim}{c}| = \sqrt{5^2 + 1^2}$$

$$= \sqrt{26} \checkmark$$

$$= \left[\begin{pmatrix} -2 \\ 5 \end{pmatrix} \cdot \frac{1}{\sqrt{26}}\begin{pmatrix} 5 \\ -1 \end{pmatrix}\right] \times \frac{1}{\sqrt{26}}\begin{pmatrix} 5 \\ -1 \end{pmatrix} \checkmark$$

$$= \frac{1}{\sqrt{26}}(-15) \times \frac{1}{\sqrt{26}}\begin{pmatrix} 5 \\ -1 \end{pmatrix} \checkmark$$

$$= \frac{-15}{26}\begin{pmatrix} 5 \\ -1 \end{pmatrix} \checkmark$$

(c) Explain the difference between a vector projection and a scalar projection [1]

scalar proj is just a magnitude whilst vector proj also has a direction.

4. (3 marks)

Ship A has a position vector of $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$ km. Relative to a second ship, B, ship A has a position vector of $\begin{pmatrix} -6 \\ 11 \end{pmatrix}$ km. Determine the exact distance of ship B from the origin.

$\overrightarrow{BA}$

$$\underset{\sim}{r}_A = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$\underset{\sim}{r}_{BA} = \begin{pmatrix} -6 \\ 11 \end{pmatrix}$$

$$\underset{\sim}{r}_{AB} = \underset{\sim}{r}_A - \underset{\sim}{r}_B$$

$$\begin{pmatrix} -6 \\ 11 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} a \\ b \end{pmatrix} \checkmark$$

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

$$= \begin{pmatrix} 5 \\ 7 \end{pmatrix} + \begin{pmatrix} 6 \\ -11 \end{pmatrix}$$

$$= \begin{pmatrix} 11 \\ -4 \end{pmatrix}$$

$$a = 11$$

$$b = -4$$

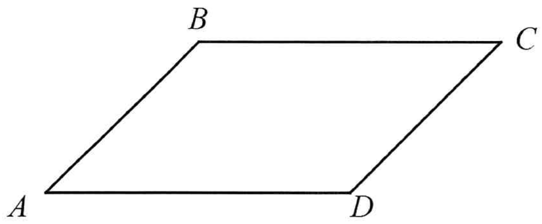
$$\therefore \underset{\sim}{r}_B = \begin{pmatrix} 11 \\ -4 \end{pmatrix} \checkmark$$

$$|\underset{\sim}{b}| = \sqrt{11^2 + 4^2}$$

$$= \sqrt{137} \checkmark$$

5. (5 marks)

Consider the figure $ABCD$ below which is a parallelogram.



Let $\overrightarrow{AB} = \underset{\sim}{b}$ and $\overrightarrow{AD} = \underset{\sim}{d}$

Prove that the diagonals AC and BD are perpendicular only when $|\underset{\sim}{b}| = |\underset{\sim}{d}|$.

$\overrightarrow{AC} = \underset{\sim}{b} + \underset{\sim}{d}$ ✓

$\overrightarrow{BD} = \underset{\sim}{d} - \underset{\sim}{b}$ ✓

$(\underset{\sim}{b} + \underset{\sim}{d}) \cdot (\underset{\sim}{d} - \underset{\sim}{b}) = 0$ for $\perp$ ✓

$\Rightarrow \underset{\sim}{b} \cdot \underset{\sim}{d} - \underset{\sim}{b} \cdot \underset{\sim}{d} + \underset{\sim}{d} \cdot \underset{\sim}{d} - \underset{\sim}{b} \cdot \underset{\sim}{b} = 0$

$\Rightarrow -b^2 + d^2 = 0$ ✓

$\Rightarrow |\underset{\sim}{b}|^2 = |\underset{\sim}{d}|^2$

$\therefore |\underset{\sim}{b}| = |\underset{\sim}{d}|$ ✓

Q.E.D

TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 2 2022

Section 2 Calculator Assumed
Vectors

STUDENT'S NAME MARKING KEY

DATE: Friday 1st April **TIME:** 20 minutes **MARKS**: 20

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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6. (3 marks)

If $\underset{\sim}{a} \cdot \underset{\sim}{b} = \underset{\sim}{a} \cdot \underset{\sim}{c}$ and $\underset{\sim}{a} \neq 0$ then what is the relationship between the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \underset{\sim}{a} \cdot \underset{\sim}{c}$$

$$\underset{\sim}{a} \cdot \underset{\sim}{b} - \underset{\sim}{a} \cdot \underset{\sim}{c} = 0$$

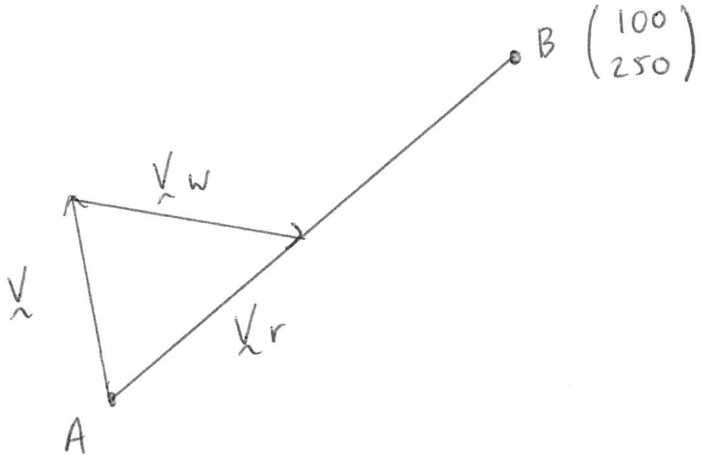
$$\underset{\sim}{a} \cdot (\underset{\sim}{b} - \underset{\sim}{c}) = 0 \quad \checkmark$$

vector $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b} - \underset{\sim}{c}$ ✓✓

7. (8 marks)

Jetties A and B are on opposite banks of a river such that $\overrightarrow{AB} = \begin{pmatrix} 100 \\ 250 \end{pmatrix}$ km. A person travelling on a jet ski can maintain a speed of 70 km/h in still air. During the trip from A to B a wind is blowing with a velocity of $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ km/h.

(a) Draw a diagram of the above situation. [2]



✓ wind

✓ general diagram.

(b) Determine the velocity vector, in component form, the jet ski rider must set so that he travels directly from jetty A to jetty B. [4]

$\underset{\sim}{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ $\quad \underset{\sim}{v}_w = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ $\quad \underset{\sim}{v}_r = \begin{pmatrix} 5+a \\ b-2 \end{pmatrix}$

$|\underset{\sim}{v}| = \sqrt{a^2+b^2}$

$70 = \sqrt{a^2+b^2}$ ✓

$t(5+a) = 100$ ✓

$t(b-2) = 250$

$a = 20.92$ ✓

$b = 66.8$

$\underset{\sim}{v} = \begin{pmatrix} 20.92 \\ 66.8 \end{pmatrix}$ ✓

or $20.92\underset{\sim}{i} + 66.8\underset{\sim}{j}$

(c) Determine the total time taken, in minutes, to travel from jetty A to B. [2]

$t = 3.86$ hrs ✓

$= 231.48$ mins ✓

8. (5 marks)

Given that $\underset{\sim}{a} = 3i + 5j$ and $\underset{\sim}{b} = xi + yj$ determine x and y if $|\underset{\sim}{b}| = \sqrt{10}$ and the acute angle between the vectors is 60°

$$\underset{\sim}{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \qquad \underset{\sim}{b} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta \qquad |\underset{\sim}{a}| = \sqrt{3^2+5^2}$$

$$3x + 5y = \sqrt{34}\sqrt{10}\cos 60^\circ \qquad = \sqrt{34} \checkmark$$

$$① \quad 3x + 5y = \frac{1}{2}\sqrt{340} \quad \checkmark$$

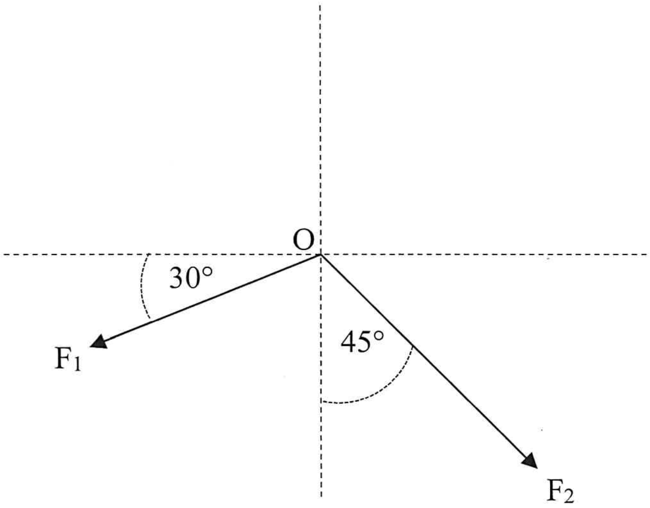
$$② \quad \sqrt{x^2+y^2} = \sqrt{10} \quad \checkmark$$

$$x = -1.53 \checkmark \qquad \text{or} \qquad x = 3.16 \checkmark$$

$$y = 2.76 \qquad\qquad y = -0.53$$

9. (4 marks)

The following diagram shows forces F_1 and F_2 acting on point O.



If $|F_1| = 1600$ N and $|F_2| = 900$ N, determine the magnitude and bearing of a single force F_3 that would keep the system in equilibrium.

$$F_1 + F_2 = \begin{pmatrix} 1600 \\ \angle(-150^\circ) \end{pmatrix} + \begin{pmatrix} 900 \\ \angle(-45^\circ) \end{pmatrix} \checkmark$$

$$= \begin{pmatrix} -749.24 \\ -1436.40 \end{pmatrix} \checkmark$$

$$\therefore F_3 = \begin{pmatrix} 749.24 \\ 1436.40 \end{pmatrix}$$

F_3 has a magnitude of 1620N ✓

F_3 is on a bearing of 028 °T ✓

<u>rads</u>

1635.86

$\angle 0.233$